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Decision-making Model for Multidimensional Analysis of Preference Based on Intuitionistic Trapezium Fuzzy Set

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(Abstract) Decision-making model for multidimensional analysis of preference on trapezoid fuzzy number distant expected values are proposed. The group decision-making problem are solved when preference and attribute are given by trapezoid fuzzy number .Its algorithm is as follows: First, define a distortion function between subjective / objective analysis of preference under B-cut set to get weighted vector of the attribute through constructing a criterion-programming model. Second, congregate the weighted normalization fuzzy decision matrices of all the decision-makers under different B-cut sets to form a total weighted normalization fuzzy decision matrix. Finally, get relative closeness S of each alternative adjustment decision and then sort by size to determine the optimal program.

Keywords: Trapezium Fuzzy Set; Multidimensional Analysis; Decision-making Mode.

1. Introduction

Hybrid aggregation operator Method [1], interactive method [2], TOPSIS method [3] are the common solutions to solve fuzzy multi-attribute decision-making problems. Among them TOPSIS method is the most classical. Literature [4] find the program's ideal solution and negative ideal solution based on fuzzy numbers' extreme points and the literature based on maximum and minimum value. For the fuzzy number distance, the literature use the vertex method that is Euclidean distance, the literature uses Hamming distance, and the literature introduces Minkowski distance. For the sorting of fuzzy numbers, the literature introduces a Rs partial order relation, literature propose the expected value TOPSIS method, the literature gets the program's ranking results through congregating ranking vectors under different α -cut sets. Currently there are fewer literatures about the research on incomplete weighted information and the program with preference trapezoid fuzzy number expected value TOPSIS method to solve the group decision-making problems. Herein, we propose a group decision-making model for multidimensional analysis of preference based on trapezoid fuzzy number distance expected values. The algorithm is as follows: First, normalize the trapezoid fuzzy number decision-making matrix, define distortion function between subjective and objective analysis of preferences under B -cut set and get weighted vector of the attribute through constructing a criterion-programming model; then congregate the weighted normalization fuzzy decision-making matrices of all the decision-makers under different B-cut set to form a total weighted normalization fuzzy decision matrix.; and then we get relative closeness Q of each optional program and the fuzzy ideal solutions based on distance expected value TOPSIS method and then sort by size to determine the optimal program. Finally, we show the effectiveness of this group decision-making model through examples.

2. Trapezium Fuzzy Set

Introduce the basic operation rules, expected value, fuzzy number distance, L / R fuzzy set concept of trapezoid fuzzy numbers.

Definition 1[5] :

Let $A=\{(a_1,b_1,c_1,d_1);\mu_A(x),\ (a_2,b_2,c_2,d_2);v_A(x)\},x\in X$, so we call A trapezoid fuzzy number, and its membership function can be defined as:

$$\mu_A(x)=\begin{cases}0 & x<a_1\\ \dfrac{x-a_1}{b_1-a_1}\mu_A & a_1\le x<b_1\\ \mu_A & b_1\le x<c_1\\ \dfrac{x-d_1}{c_1-d_1}\mu_A & c_1\le x<d_1\\ 0 & x\ge d_1\end{cases} \quad (1)$$

$$v_A(x)=\begin{cases}0 & x<a_2\\ \dfrac{b_2-x+v_A(x-a_2)}{b_2-a_2} & a_2\le x<b_2\\ v_A & b_2\le x<c_2\\ \dfrac{x-c_2+v_A(d_2-x)}{d_2-c_2} & c_2\le x<d_2\\ 0 & x\ge d_2\end{cases} \quad (2)$$

Definition 2[6] :

Let $A=\{a_1,b_1,c_1,d_1\}$ and $B=\{a_2,b_2,c_2,d_2\}$ be two trapezoid fuzzy numbers, so the basic operation rules of addition, subtraction and numerical multiplication of fuzzy numbers are as follows:

（1） addition：

$$A+B=\{a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2\} \quad (3)$$

（2） subtraction：

$$A-B=\{a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2\} \quad (4)$$

（3） numerical multiplication：

$$\lambda\times A=\{\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1\} \quad \lambda>0 \quad (5)$$

Definition 3[7]：Let expected value of trapezoid fuzzy number $A=\{a,b,c,d\}$ be：

$$E(A)=\frac{1}{4}(a+b+c+d) \quad (6)$$

Define 4 ：Let the distance between two trapezoid fuzzy numbers $A=\{a_1,b_1,c_1,d_1\}$ and $B=\{a_2,b_2,c_2,d_2\}$ be：

$$d(A,B)=E(A,B)=\frac{1}{4}(|a_1-d_2|+|b_1-c_2|+|c_1-b_2|+|d_1-a_2|) \quad (7)$$

Definition 5：Let $A=\{a,b,c,d\}$ be a trapezoid fuzzy number，so the set formed by all x satisfying $\mu_A(x)\ge\beta$ is called B-cut set of A:

$$A^L(\beta)=b\times\beta+(1-\beta)\times a$$

It is represents left fuzzy set（L fuzzy set for short）.

$$A^R(\beta)=c*\beta+(1-\beta)\times d$$

It is represents the right fuzzy set (R fuzzy set for short).

3. Trapezoid fuzzy number group decision-making model

Assume that $S=\{s_1,s_2,\ldots s_m\}$ is fuzzy number multi-attribute group decision-making program set $G=\{G_1,G_2,\ldots G_n,\}$ is property set，$P=\{P_1,P_2,\ldots P_t\}$ is decision-makers set，$X^{(k)}=(x_{ij}^{(k)})_{m\times n}$ is trapezoid fuzzy number decision-making matrix of decision-maker P_k to decision-making program set S with property G, $T^{(k)}=(t_{ij}^{(k)})_{m\times n}$ is preference trapezoid fuzzy number decision-making matrix of decision-maker P_k to decision-making program set S with property G，the weight of decision-making P_k is λ_k （$k=1,2,\ldots t$）,the weight vector of the decision-making P_k to the property G is $\omega=(\omega_1,\omega_2,\text{K}\,\omega_n)$, H is a weight collection of a known part with certain weight information and $\omega\in H$ .The goal of group decision-making is to select a program as the optimal program which is corresponded to the maximum value of relative closeness δ_i of fuzzy ideal solution from m optional programs. The following introduces a group decision-making

model for multidimensional analysis of preference based on trapezoid fuzzy number expected values and its implementation steps:

Step (1): normalize trapezoid fuzzy number original decision-making matrix

To eliminate the impact of different physical dimensions on the decision-making result, fuzzy decision-making matrix must be normalized and the property of efficiency (denoted as L1) and cost (denoted as L2) must be respectively treated according to the formula (8) and (9) to form normalization fuzzy decision-making matrix.

$$r_{ij}^{(k)} = \left(\frac{a_{ij}^{(k)}}{u_{ij}^{(k)}}, \frac{b_{ij}^{(k)}}{u_{ij}^{(k)}}, \frac{c_{ij}^{(k)}}{u_{ij}^{(k)}}, \frac{d_{ij}^{(k)}}{u_{ij}^{(k)}}\right) \quad j \in L1 \qquad (8)$$

$$r_{ij}^{(k)} = \left(\frac{v_{ij}^{(k)}}{d_{ij}^{(k)}}, \frac{v_{ij}^{(k)}}{c_{ij}^{(k)}}, \frac{v_{ij}^{(k)}}{b_{ij}^{(k)}}, \frac{v_{ij}^{(k)}}{a_{ij}^{(k)}}\right) \quad j \in L2 \qquad (9)$$

Where： $u_{ij}^{(k)} = \max\{d_{ij}^{(k)}\} j \in L1$ and $v_{ij}^{(k)} = \min\{a_{ij}^{(k)}\} j \in L2$.

Step (2): set up left and right weighted distortion function model of subjective and objective preference;

Let the trapezoid fuzzy vector of subjective preferences be $t_i^{(k)} = \{t_{i1}, t_{i2}, t_{i3}, t_{i4}\}$, the trapezoid fuzzy vector of objective be $r_i^{(k)} = \{r_{i1}, r_{i2}, r_{i3}, r_{i4}\}$, L/R fuzzy matrices of subjective and objective preferences under the β_L -cut set can be respectively expressed as $\tilde{t}_{ij}^{(k)}(\beta_L) = [t_{ij}^L(\beta_L), t_{ij}^R(\beta_L)]$ and $\tilde{r}_{ij}^{(k)}(\beta_L) = [r_{ij}^L(\beta_L), r_{ij}^R(\beta_L)]$, so left and right weighted distortion function model[8]-[11] of subjective and objective preferences are:

$$f_{ij}^{(k)}(\beta_L) = \left| r_{ij}^L(\beta_L) - t_{ij}^L(\beta_L) \right| \times \omega_j \qquad (10)$$

$$g_{ij}^{(k)}(\beta_L) = \left| r_{ij}^R(\beta_L) - t_{ij}^R(\beta_L) \right| \times \omega_j \qquad (11)$$

Where：$i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$ and $k = 1,2,\ldots,t$.

Step（3）：Optimization Model

Considering the rationality of the decision-making, the selection of the weight vector should make the decision-maker have the smallest total distortion of subjective and objective preferences under the cut set β_L .So we need to build up an optimization model 1:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} (\lambda_1 f_{ij}^{(k)}(\beta_L) + \lambda_2 g_{ij}^{(k)}(\beta_L))$$

$$s.t. \omega \in H, \omega_j \geq 0, \sum_{j=1}^{n} \omega_j = 1$$

Where: λ_1 and λ_2 are respectively weight coefficients of left and right distortion.

Considering that each distortion function is fair competition, has not any preference relation and has the hope that those expectations are respectively zero, we can generally make $\lambda_1 = \lambda_2$ =1. Therefore, the model 1 can be transformed into the criterion-programming mode 2:

$$\min L = \sum_{i=1}^{m} \sum_{j=1}^{n} [e_{ij}^{+}(\beta_L) + e_{ij}^{-}(\beta_L) + d_{ij}^{+}(\beta_L) + d_{ij}^{-}(\beta_L)]$$

$$s.t. \begin{cases} r_{ij}^L(\beta_L)\omega_j - t_{ij}^L(\beta_L)\omega_j + e_{ij}^{+} - e_{ij}^{-} = 0; \\ r_{ij}^R(\beta_L)\omega_j - t_{ij}^R(\beta_L)\omega_j + d_{ij}^{+} - d_{ij}^{-} = 0; \\ e_{ij}^{+}, e_{ij}^{-}, d_{ij}^{+}, d_{ij}^{-} \geq 0; \\ \omega \in H, \omega_j \geq 0, \sum_{j=1}^{n} \omega_j = 1 \end{cases}$$

We can get the property weight vector $\omega^{(k)}(\beta_L) = \{\omega_1^{(k)}(\beta_L), \omega_2^{(k)}(\beta_L), \ldots \omega_m^{(k)}(\beta_L)\}$ under β_L -cut set based on Lingo software programming to solve the model 2 [13]-[15].

Step (4): Congregate the weighted normalization fuzzy decision-making matrices Congregate the weighted normalization fuzzy decision-making matrices of decision-maker P_k based on the weighted arithmetic average operator to form the total weighted normalization fuzzy decision-making matrix $\tilde{R}_L^{(k)} = (\tilde{r}_{i1}^{(k)}, \tilde{r}_{i2}^{(k)} \ldots \tilde{r}_{in}^{(k)})$, where $\tilde{r}_{ij}^{(k)} = r_{ij}^{(k)} \times \omega_j^{(k)}(\beta_L)$. It is not conducive to calculating and solving the planning model because of the

parameter β_L in it. So first , generally take $\beta_L = \frac{s}{p}$ ($s = 1,2,\dots n$,P is calculation accuracy) into the model 2 to calculate, then calculate the arithmetic mean $\overline{R}^{(k)} = \frac{1}{s+1}\sum_{L=0}^{s}\tilde{R}_L^{(k)}$ of the decision-making matrices under all the β_L -cut sets, at last congregate $\overline{R}^{(k)}$ of t decision-makers to form the total weighted normalization fuzzy decision matrix $\overline{R} = \sum_{k=1}^{t}\vec{R}^{(k)} \times \lambda_k = \{\tilde{r}_{i1}, \tilde{r}_{i2}, \dots, \tilde{r}_{in}\}$.

Step (5): seek fuzzy ideal solution and fuzzy negative ideal solution

In the total weighted normalization fuzzy decision matrix, calculate trapezoid fuzzy number expected value based on the formula (5) and seek fuzzy ideal solution $R^+ = (r_1^{\ +}, r_2^{\ +}, \dots r_n^{\ +})$ and fuzzy negative ideal solution $R^- = (r_1^{\ -}, r_2^{\ -}, \dots r_n^{\ -})$ of the program. Where $r_j^+ = \{\tilde{r}_{ij} \,|\, \max E(\tilde{r}_{ij})\}$ and $r_j^- = \{\tilde{r}_{ij} \,|\, \min E(\tilde{r}_{ij})\}$.

Step (6): Sort by distance expected value Respectively calculate distance value (D^+ , D^-) of each program to fuzzy ideal solution and fuzzy negative ideal solution according to the calculation formula (6), (12) and (13) of the trapezoid fuzzy number distance expected value.

$$D^+ = \sum_{j=1}^{n} d(\tilde{r}_{ij}, \tilde{r}_j^{\,+}) \quad i \in m \qquad (12)$$

$$D^- = \sum_{j=1}^{n} d(\tilde{r}_{ij}, \tilde{r}_j^{\,-}) \quad i \in m \qquad (13)$$

Then get relative closeness δ_i of each program and the fuzzy ideal solution according to the formula (14) and afterwards sort by its size to determine the optimal program.

$$\delta_i = \frac{D^-}{D^- + D^+} \qquad (14)$$

The program with the bigger relative closeness indicates it is closer to the fuzzy ideal solution. And the program with the maximum relative closeness value is the optimal program.

4. Example Analysis

Extend the example in literature according to the model proposed in this paper to show the feasibility and correctness of using trapezoid fuzzy number expected value TOPSIS model to make group decision.

A school have developed three optional procurement schemes according to the need of information construction, and asked respectively three decision-makers assessing hazily on three attribute indexes: production cost, product performance and after-sales service. Afterwards form the following decision-making matrix expressed by trapezoid fuzzy number:

$$X^1 = \begin{bmatrix} (2.27,2.14,4.20,4.35)(3.74,4.93,3.02,3.26)(4.53,4.88,3.07,5.39) \\ (3.42,4.76,4.89,4.05)(4.70,3.90,4.09,4.27)(5.38,5.82,4.03,5.20) \\ (4.63,6.05,5.14,4.38)(5.74,4.96,4.08,5.26)(3.57,4.86,5.97,3.29) \end{bmatrix}$$

$$X^2 = \begin{bmatrix} (3.57,4.16,4.21,4.25)(3.54,3.73,3.32,4.36)(4.43,3.78,4.27,3.49) \\ (4.42,5.56,4.98,5.15)(4.50,4.56,4.19,5.17)(3.28,4.62,5.13,4.23) \\ (5.73,6.15,5.24,6.28)(5.76,5.86,5.18,5.36)(4.47,5.86,5.64,4.47) \end{bmatrix}$$

$$X^3 = \begin{bmatrix} (3.57,4.24,4.22,4.33)(5.64,3.73,3.22,4.46)(3.56,4.68,5.12,3.31) \\ (4.42,5.76,5.87,5.15)(6.74,4.70,4.39,5.47)(4.58,5.82,4.12,4.21) \\ (5.73,6.25,6.34,5.18)(6.75,5.96,5.28,4.36)(4.47,3.86,3.65,5.31) \end{bmatrix}$$

Because of the different subjective preferences and the different actual procurement experience of each individual decision-maker, they have different subjective preference values (expressed by trapezoid fuzzy numbers) of these three optional programs, so preference trapezoid fuzzy matrices are respectively:

$$T^1 = \begin{bmatrix} (0.81,0.59,0.64,0.72) \\ (0.74,0.56,0.66,0.79) \\ (0.86,0.51,0.62,0.81) \end{bmatrix}$$

$$T^2 = \begin{bmatrix} (0.73,0.51,0.51,0.61) \\ (0.81,0.63,0.69,0.72) \\ (0.81,0.60,0.79,0.72) \end{bmatrix}$$

$$T^3 = \begin{bmatrix} (0.51,0.74,0.68,0.80) \\ (0.63,0.80,0.71,0.87) \\ (0.74,0.89,0.51,0.76) \end{bmatrix}$$

The known value ranges of these three attributes are as follows:

$0.36 \le \omega_1 \le 0.47$ $0.23 \le \omega_2 \le 0.45$ and $0.13 \le \omega_3 \le 0.35$,

expressed by $H = \{\omega = \{\omega_1, \omega_2, \omega_3\}\}$. Assume the weights of these three decision-makers are $\lambda_1 = 0.3, \lambda_2 = 0.4, \lambda_3 = 0.3$.

Try to analyze which optional scheme is the optimal one.

First, respectively normalize the fuzzy trapezoid decision-making matrices X^1, X^2, X^3 of these three decision-makers based on the formula (8) or (9) according to the group decision-making steps from (1) to (3). And translate the decision-making matrix after normalizing into L/R fuzzy matrix when β_L =0.

Get the attribute weight vectors $\omega^1(0) = \{0.32,0.37,0.21\}$, $\omega^2(0) = \{0.32,0.23,0.24\}$,and $\omega^3(0) = \{0.33,0.26,0.21\}$ corresponded to the decision-makers when β_L =0 based on Lingo software programming.

Second, congregate total weighted normalization fuzzy decision-making matrices when β_L =0, then calculate each weight vector when β_L is separately 0.2, 0.4, 0.6, 0.8 and 1.0, at last congregate weighted normalization fuzzy decision-making matrices of these three decision-makers (because of space limitations, the middle calculation process is omitted) to form the total weighted normalization fuzzy decision matrix according to the step (4):

$$\tilde{R} = \begin{bmatrix} (0.21,0.32,0.67,0.89)(0.54,0.65,0.87,0.71)(0.56,0.63,0.64,0.71) \\ (0.31,0.42,0.34,0.87)(0.46,0.73,0.52,0.85)(0.75,0.83,0.76,0.78) \\ (0.41,0.23,0.56,0.67)(0.65,0.73,0.82,0.65)(0.63,0.76,0.63,0.67) \end{bmatrix}$$

On this basis, seek fuzzy ideal solution R^+ and negative ideal solution R^- according to step (5).

$$R^+ = \left[(0.21,0.32,0.67,0.89)(0.65,0.73,0.82,0.65)(0.75,0.83,0.76,0.78)\right]$$

$$R^- = \left[(0.41,0.23,0.56,0.67)(0.46,0.73,0.52,0.85)(0.56,0.63,0.64,0.71)\right]$$

At last calculate distance value D^+ from each optional program to fuzzy ideal solution according to the formula（12）in step (6)

$D_1^+ = 0.745$, $D_2^+ = 0.789$, $D_3^+ = 0.805$

Calculate distance value D^- from each optional program to fuzzy ideal negative solution according to the formula （13）in step (6)

$D_1^- = 1.21$, $D_2^- = 0.855$, $D_3^- = 0.728$

Calculate relative closeness δ_i from the optional program to the fuzzy ideal solutions according to the formula (14) : $\delta_1 = 0.59$, $\delta_2 = 0.46$, $\delta_3 = 0.51$,Sort by the size of δ_i from big to small, the result is $\delta_1 > \delta_3 > \delta_2$. The advanced and disdained sorting of the optional programs is $s_1 > s_3 > s_2$ .Therefore, program s1 is the optimal program. The result considers the same with the literature.

5. Conclusion

The innovation in this paper is proposing a new TOPSIS method based on trapezoid fuzzy number expected value. This solves the multi-dimensional preference group decision-making problems that preferences and property value are both trapezoid fuzzy number multi-dimensional preference group decision-making problems. The algorithm is as follows: First, normalize the trapezoid fuzzy number decision-making matrix, define distortion function between subjective and objective analysis of preferences under B-cut set and get weighted vector of the attribute through constructing a criterion-programming model; then congregate the weighted normalization fuzzy decision-making matrices of all the decision-makers under different B-cut set to form a total weighted normalization fuzzy decision matrix.; and then we get relative closeness S of each optional program and the fuzzy ideal solutions based on distance expected value TOPSIS method and then sort by size to determine the optimal

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